# Stability, Competitive Equilibrium, and Full Substitutability

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(Based on work with S.D. Kominers, A. Nichifor, M. Ostrovsky, and A. Westkamp)

# Background

In matching, auction theory, and exchange economies with indivisible goods, some form of "substitutability" is usually essential for guaranteeing the existence of equilibria (if we allow arbitrary "simple" preferences).

Matching: Kelso & Crawford (1982); Roth (1984); Hatfield & Milgrom (2005); Ostrovsky (2008); Hatfield & Kominers (2012)...

Auctions: Milgrom (2000); Ausubel & Milgrom (2006)...

Exchange: Bikhchandani & Mamer (1997); Gul & Stacchetti (1999, 2000); Sun & Yang (2006, 2009)...

# Background

And underpins a number of high-profile applications:

- ⋆ Spectrum auctions

  (Ausubel, 2006)

- ⋆ Oil and natural gas supply-chains

  (Ostrovsky, 2008)

- ⋆ "Swap" deals in exchange markets

  (Milgrom, 2009)

- ⋆ Natural gas auctions

  (Milgrom & Strulovici, 2009)

- ⋆ Securities auctions

  (Klemperer, 2010; Baldwin–Klemperer, 2014, 2015)

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- How does the definition deal with constraints?

We show that all of these works use the same notion of substitutability.

# Overview of Results

Matching:

Roth (1984) — (S)

Ostrovsky (2008), Hatfield & Kominers (2012) — (SSS)+(CSC)

Hatfield et al. (2013) — (FS)

In fully general trading networks:

1. choice-language full substitutability,*

# Overview of Results

Exchange:

Gul & Stachetti (1999) — (GS), (SI), (NC); (SM)

Sun & Yang (2006) — (GSC)

Baldwin & Klemperer (2019) — (GS), (OS)

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2. demand-language full substitutability,*

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Auction Theory:

Milgrom–Strulovici (2009) — (WS)/(SS); (SM)

Gul–Stachetti (2000) — (GS), (SI), (NC); (SM)

Milgrom (2000) — (SM)

Ausubel–Milgrom (2002), — (S); (SM)

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In fully general trading networks:

1. choice-language full substitutability,*
2. demand-language full substitutability,*
3. indicator-language full substitutability,*
4. submodularity of indirect utility,
5. object-language full substitutability,
6. the single improvement property,
7. the no-complementarities condition, and

# Overview of Results

OR / Math:

Fujishige–Yang (2003) — (GS), ($M^\natural$–c)

Shioura–Tamura (2015) — (GS), ($M^\natural$–c), (SS); (SM); ($M^\natural$–EXC) (PRJ-GS), (SWGS), (GS+LAD)

Paes Leme (2014) — (GS), ($M^\natural$–c), ...

Baldwin–Klemperer (2014) — (GS),($\mathcal{D}^n_{OS}$),(TH)

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are all equivalent.

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**Theorem (Transformations)**

*Full substitutability is preserved under*

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## Theorem (Existence)

*When each agent's preferences are substitutable, a competitive equilibrium exists.*

## Theorem (Solution Concept Equivalence)

*When each agent's preferences are substitutable, an outcome is*

$$\textit{CE} \iff \textit{Stable} \iff \textit{Chain stable}.$$

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  - $\Psi_{\rightarrow i} \equiv \{\psi \in \Psi : b(\psi) = i\}$.
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- An **arrangement** is a pair $[\Psi; p]$, where $\Psi \subseteq \Omega$ and $p \in \mathbb{R}^{\Omega}$.
- Set of **contracts** $X \equiv \Omega \times \mathbb{R}$
  - each contract $x \in X$ is a pair $(\omega, p_\omega)$
  - $\tau(Y) \subseteq \Omega \sim$ set of trades in contract set $Y \subseteq X$

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  - each contract $x \in X$ is a pair $(\omega, p_\omega)$
  - $\tau(Y) \subseteq \Omega \sim$ set of trades in contract set $Y \subseteq X$
- A **(feasible) outcome** is a set of contracts $A \subseteq X$ which uniquely prices each trade in $A$.

# The Setting: Demand

- Each agent $i$ has quasilinear utility over arrangements:

$$U_i([\Psi; p]) = u_i(\Psi_i) - \sum_{\psi \in \Psi_{\to i}} p_\psi + \sum_{\psi \in \Psi_{i \to}} p_\psi.$$

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- For any price vector $p \in \mathbb{R}^\Omega$, the **demand** of $i$ is

$$D_i(p) = \arg\max_{\Psi \subseteq \Omega_i} U_i([\Psi; p]).$$

- For any set of contracts $Y \subseteq X$, the **choice** of $i$ is $C_i(Y) = \arg\max_{Z \subseteq Y_i} U_i(Z)$.

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2. $u_i(\varnothing) \in \mathbb{R}$.
3. **Full substitutability**...

# Choice-Language Full Substitutability

## Definition

The preferences of agent $i$ are **fully substitutable** in **choice language** if for all sets of contracts $Y, Z \subseteq X_i$ such that $|C_i(Z)| = |C_i(Y)| = 1$,

1. if $Y_{i\to} = Z_{i\to}$, and $Y_{\to i} \subseteq Z_{\to i}$,
   then for $Y^* \in C_i(Y)$ and $Z^* \in C_i(Z)$,
   we have $(Y_{\to i} \setminus Y^*_{\to i}) \subseteq (Z_{\to i} \setminus Z^*_{\to i})$ and $Y^*_{i\to} \subseteq Z^*_{i\to}$;
2. if $Y_{\to i} = Z_{\to i}$, and $Y_{i\to} \subseteq Z_{i\to}$,
   then for $Y^* \in C_i(Y)$ and $Z^* \in C_i(Z)$,
   we have $(Y_{i\to} \setminus Y^*_{i\to}) \subseteq (Z_{i\to} \setminus Z^*_{i\to})$ and $Y^*_{\to i} \subseteq Z^*_{\to i}$.

# Demand-Language Full Substitutability

## Definition

The preferences of agent $i$ are **fully substitutable** in **demand language** if for all $p, p' \in \mathbb{R}^{\Omega}$ such that $|D_i(p)| = |D_i(p')| = 1$,

1. if $p_\omega = p'_\omega$ for all $\omega \in \Omega_{i\to}$, and $p_\omega \geq p'_\omega$ for all $\omega \in \Omega_{\to i}$, then for the unique $\Psi \in D_i(p)$ and $\Psi' \in D_i(p')$, we have
$$\{\omega \in \Psi'_{\to i} : p_\omega = p'_\omega\} \subseteq \Psi_{\to i}, \quad \Psi_{i\to} \subseteq \Psi'_{i\to};$$
2. if $p_\omega = p'_\omega$ for all $\omega \in \Omega_{\to i}$, and $p_\omega \leq p'_\omega$ for all $\omega \in \Omega_{i\to}$, then for the unique $\Psi \in D_i(p)$ and $\Psi' \in D_i(p')$, we have
$$\{\omega \in \Psi'_{i\to} : p_\omega = p'_\omega\} \subseteq \Psi_{i\to}, \quad \Psi_{\to i} \subseteq \Psi'_{\to i}.$$

# Indicator-Language Full Substitutability

$$e_{i,\omega}(\Psi) = \begin{cases} 1 & \omega \in \Psi_{\to i} \\ -1 & \omega \in \Psi_{i\to} \\ 0 & \text{otherwise} \end{cases}$$

## Definition

The preferences of agent $i$ are **fully substitutable** in **indicator language** if for all price vectors $p, p' \in \mathbb{R}^{\Omega}$ such that $|D_i(p)| = |D_i(p')| = 1$ and $p \leq p'$, for $\Psi \in D_i(p)$ and $\Psi' \in D_i(p')$, we have

$$e_{i,\omega}(\Psi) \leq e_{i,\omega}(\Psi')$$

for each $\omega \in \Omega_i$ such that $p_\omega = p'_\omega$.

# Equivalence

**Theorem**

*Choice-Language Full Substitutability* $\iff$ *Demand-Language Full Substitutability* $\iff$ *Indicator-Language Full Substitutability.*

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Allowing for definitions that look at price vectors for which the choice function is not single-valued is straightforward.

# Submodular Indirect Utility Functions

## Definition

The **indirect utility function** of agent $i$,

$$V_i(p) \equiv \max_{\Psi \subseteq \Omega_i} \{ U_i([\Psi; p]) \},$$

is **submodular** if, for all price vectors $p, \bar{p} \in \mathbb{R}^{\Omega}$, we have that

$$V_i(p \wedge \bar{p}) + V_i(p \vee \bar{p}) \leq V_i(p) + V_i(\bar{p}).$$

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## Theorem

*A utility function is fully substitutable if and only if its associated indirect utility function is submodular.*

# "Underlying Objects"

Let

$$\mathfrak{o}_i(\Psi) \equiv \{\mathfrak{o}(\omega) : \omega \in \Psi_{\to i}\} \cup \{\mathfrak{o}(\omega) : \omega \in \Omega_i \setminus \Psi_{i\to}\}.$$

# "Underlying Objects"

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A utility function is object-language fully substitutable if the induced utility function over objects is grossly substitutable (in the sense of Kelso & Crawford (1982)).

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*A utility function is fully substitutable if and only if it is object-language fully substitutable.*

# The Single Improvement Property

## Classic Idea

When $i$ does not have an optimal bundle, he can make himself better off by adding a single item, dropping a single item, or both.

# The Single Improvement Property

## Definition

The preferences of agent $i$ have the **single improvement property** if whenever $i$ holds a suboptimal bundle of trades $\Psi$, he can be made be better off by

1. obtaining one object not currently held (either by making a new purchase, i.e., adding a trade in $\Omega_{\to i} \setminus \Psi$, or by canceling a sale, i.e., removing a trade in $\Psi_{i\to}$),
2. relinquishing one object currently held (either by canceling a purchase, i.e., removing a trade in $\Psi_{\to i}$, or by making a new sale, i.e., adding a trade in $\Omega_{i\to} \setminus \Psi$), or
3. both obtaining one object not currently held and relinquishing one object currently held.

Fully substitutable choice functions satisfy the single improvement property (Gul & Stacchetti (1999) + object-language full substitutability).

# Trade Endowments (I)

Suppose an agent $i$ is endowed with the right to execute trades in the set $\Phi \subseteq \Omega_i$ at prices $p_\Phi$. Let

$$\hat{u}_i^{(\Phi, p_\Phi)}(\Psi) \equiv \max_{\Xi \subseteq \Phi} \left\{ u_i(\Psi \cup \Xi) + \sum_{\xi \in \Xi_{\to i}} p_\xi - \sum_{\xi \in \Xi_{\to i}} p_\xi \right\}$$

be the induced valuation over trades in $\Omega \setminus \Phi$.

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**Theorem**

*If the preferences of agent $i$ are fully substitutable, then the preferences induced by $\hat{u}_i^{(\Phi,p_\Phi)}$ are, as well.*

# Trade Endowments (II)

The indirect utility function for $\hat{u}_i^{(\Phi,p_\Phi)}$ is given by

$$\begin{aligned}
\hat{V}_i^{(\Phi,p_\Phi)}(p_{\Omega\setminus\Phi}) &\equiv \max_{\Psi\subseteq\Omega\setminus\Phi}\left\{\max_{\Xi\subseteq\Phi}\left\{u_i(\Psi\cup\Xi)+\sum_{\xi\in\Xi_{\to i}}p_\xi-\sum_{\xi\in\Xi_{\to i}}p_\xi\right\}+\sum_{\psi\in\Psi_{\to i}}p_\xi-\sum_{\psi\in\Psi_{\to i}}p_\xi\right. \\
&= \max_{\Psi\subseteq\Omega\setminus\Phi}\left\{\max_{\Xi\subseteq\Phi}\left\{u_i(\Psi\cup\Xi)+\sum_{\lambda\in\Xi_{\to i}\cup\Psi_{\to i}}p_\lambda-\sum_{\lambda\in\Xi_{i\to}\cup\Psi_{i\to}}p_\lambda\right\}\right\} \\
&= \max_{\Lambda\subseteq\Omega}\left\{u_i(\Lambda)+\sum_{\lambda\in\Lambda_{\to i}}p_\lambda-\sum_{\lambda\in\Lambda_{i\to}}p_\lambda\right\} \\
&= V_i(p_{\Omega\setminus\Phi},p_\Phi).
\end{aligned}$$

$V_i(p)$ is submodular over $\mathbb{R}^\Omega \Rightarrow \hat{V}_i^{(\Phi,p_\Phi)}(p_{\Omega\setminus\Phi})$ is submodular over $\mathbb{R}^{\Omega\setminus\Phi}$.

# Trade Obligations

Suppose an agent $i$ is obliged to execute trades in the set $\Phi \subseteq \Omega_i$ at prices $p_\Phi$. Let

$$\tilde{u}_i^{(\Phi, p_\Phi)}(\Psi) \equiv u_i(\Psi \cup \Phi) + \sum_{\varphi \in \Phi_{i\to}} p_\varphi - \sum_{\varphi \in \Phi_{\to i}} p_\varphi$$

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be the induced valuation over trades in $\Omega \setminus \Phi$.

**Theorem**

*If the preferences of agent $i$ are fully substitutable, then the preferences induced by $\tilde{u}_i^{(\Phi, p_\Phi)}$ are as well (so long as $u_i(\Phi) \neq -\infty$).*

# Mergers

For $J \subseteq I$, we denote $\Omega^J \equiv \{\omega \in \Omega : \{b(\omega), s(\omega)\} \subseteq J\}$. The **convolution** of the valuation functions $\{u_j\}_{j \in J}$ is

$$u_J(\Psi) \equiv \max_{\Phi \subseteq \Omega^J} \left\{ \sum_{j \in J} u_j(\Psi \cup \Phi) \right\},$$

defined over $\Psi \subseteq \Omega \setminus \Omega^J$.

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defined over $\Psi \subseteq \Omega \setminus \Omega^J$.

**Theorem**

*If the preferences of each $j \in J$ are fully substitutable, then the preferences induced by the convolution $u_J$ are, as well.*

# Transformations: Limited Liability

Take an arbitrary set of trades $\Phi \subseteq \Omega_i$, and for every trade $\varphi \in \Phi$, pick $\Pi_\varphi \in \mathbb{R}$—the penalty for reneging on trade $\varphi$. Define the modified valuation function $\hat{u}_i$ as

$$\hat{u}_i(\Psi) \equiv \max_{\Xi \subseteq \Psi \cap \Phi} \left\{ u_i(\Psi \setminus \Xi) - \sum_{\varphi \in \Xi} \Pi_\varphi \right\}.$$

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$$\hat{u}_i(\Psi) \equiv \max_{\Xi \subseteq \Psi \cap \Phi} \left\{ u_i(\Psi \setminus \Xi) - \sum_{\varphi \in \Xi} \Pi_\varphi \right\}.$$

**Theorem**

*For any $\Phi \subseteq \Omega_i$ and $\Pi_\Phi \in \mathbb{R}^\Phi$, if the $u^i$ is fully substitutable, then $\hat{u}_i$ is fully substitutable.*

# Implications of Full Substitutability

**Theorem (Hatfield et al. (2013, 2020))**

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**Theorem (Hatfield et al. (2013, 2020))**

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- It is also computationally "easy" to find competitive equilibria in this setting (Candogan et al., 2018)

# Open Questions

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   - Much less seems to be known here.

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   - Do these operations have economic meaning?
2. Can we characterize substitutability in settings *without* transfers?
   - Much less seems to be known here.
3. Without transfers, all of our nice results require that the network structure be acyclic—why?